Printed Pages : 8 **AS-101**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1106 Roll No.

B. Tech.

(Semester-I) Theory Examination, 2012-13

ENGINEERING MATHEMATICS-I

Time : 3 Hours] *[Total Marks : 100*

Note : Attempt questions from each Section as per instructions. The symbols have their usual meaning.

Section-A

Attempt *all* parts of this question. Each part carries 2 marks. 2×10=20

1. (a) If $y = x^2 . \exp(2x)$, determine $(y_n)_0$.

(b) Find the radius of curvature for the curve $s = \log(\tan\psi + \sec\psi) + \tan\psi\sec\psi$, where ψ is the angle which the tangent at any pont to the curve makes with the x-axis.

(c) If $u(x, y) = \left(\sqrt{x} + \sqrt{y}\right)^5$, find the value of

$$\left(x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}\right).$$

(d) The formula, $V = kr^4$, says that the volume V of fluid flowing through a small pipe or tube in a unit of time at a fixed pressure is a constant times the fourth power of the tube's radius r. How will a 10% increase in r affect V ?

(e) Use Beta function to evaluate :

$$\int_0^{\infty} \frac{x^8(1-x^6)}{(1+x)^{24}}\,dx$$

(f) Changing the order of integration in the double integral :

$$I = \int_0^8 \int_{\pi/4}^2 f(x, y)\,dx\,dy \text{ leads to}$$

$$I = \int_r^s \int_p^q f(x, y)\,dy\,dx \text{ say,}$$

What is p ?

(g) If $\vec{F} = \frac{\vec{r}}{r^3}$, find $\operatorname{curl} \vec{F}$.

(h) Using Green's theorem, evaluate the integral :

$$\oint_C (xy\,dy - y^2 dx),$$

where C is the square cut from the first quadrant by the lines $x = 1$, $y = 1$.

(i) If $\alpha_1, \alpha_2, \alpha_3, \ldots\ldots, \alpha_n$ are the charcteristics roots of the n-square matrix A and k is a scalar, prove that the characteristic roots of $[A - kI]$ are

$$\alpha_1 - k, \alpha_2 - k, \alpha_3 - k, \ldots\ldots, \alpha_n - k.$$

(j) Explain the working rule to find the inverse of a matrix A by elementary row or column transformations.

Section-B

Attempt any *three* parts of this question. Each part carries 10 marks. 10×3=30

2. (a) Find the values of a and b such that the expansion of $\log(1+x) - \dfrac{x(1+ax)}{(1+bx)}$ in ascending powers of x begins with the term x^4 and hence find this term.

(b) Locate the stationary points of :

$$x^4 + y^4 - 2x^2 + 4xy - 2y^2$$

and determine their nature.

(c) Evaluate :

$$\int_R \int (x-y)^4 . \exp(x+y) . dx\, dy,$$

where R is the square in the xy-plane with vertices at $(1, 0), (2, 1), (1, 2)$ and $(0, 1)$.

(d) Verify the Gauss divergence theorem for :

$$\vec{F} = (x^2 - yz)\hat{i} + (y^2 - xz)\hat{j} + (z^2 - xy)\hat{k}$$

taken over the rectangular parallelopiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c.$

(e) Diagonalize the following matrix A :

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}.$$

Section-C

Attempt *all* questions of this Section. Attempt any *two* parts from each question. Each question carries 10 marks. $10 \times 5 = 50$

3. (a) If $y = \sin[\log(x^2 + 2x + 1)]$, prove that :

$$(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0.$$

(b) Trace the curve $y = x(x^2 - 1)$.

(c) Show that the radii of curvature of the curve $y^2 = \dfrac{x^2(a+x)}{(a-x)}$ at the origin are $\pm a\sqrt{2}$.

4. (a) The rate of flow Q of water per second over the sharp-edged notch of length l, the height of the general level of the water above the bottom of the notch being h, is given by the formula $Q = c\left(l - \frac{h}{5}\right)h^{3/2}$, where c is a constant. Show that for small error δh in the measurement of h, the error δQ in Q is :

$$\frac{1}{2}c(3l - h)h^{1/2}.\ \delta h.$$

(b) Show that the envelope of the family of parabolas.

$$\left(\frac{x}{a}\right)^{1/2} + \left(\frac{y}{b}\right)^{1/2} = 1,$$

under the condition $ab = c^2$ (a, b and c are constants), is a hyperbola whose asymptotes coincides the axes.

(c) Expand $f(x, y) = y^x$ about (1, 1) up to second degree terms and hence evaluate $(1.02)^{1.03}$.

5. (a) Find the mass of the solid bounded by the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ and the coordinate planes where the density at any point $P(x, y, z)$ is $kxyz$, where k is a constant.

(b) Prove that :

$$\Gamma m \, \Gamma\left(m+\frac{1}{2}\right)=\frac{\sqrt{\pi}\,\Gamma(2m)}{(2)^{2m-1}}.$$

(c) Evaluate :

$$\int_0^{\infty}\int_0^{x} x.\exp\left(-\frac{x^2}{y}\right).dx\,dy.$$

6. (a) A particle moves along a plane curve such that its linear velocity is perpendicular to the radius vector. Show that the path of the particle is a circle.

(b) Find the directional derivative of v^2, where $\vec{v}=xy^2\hat{i}+zy^2\hat{j}+xz^2\hat{k}$ at the point (2, 0, 3) in the direction of the outward normal to the sphere $x^2+y^2+z^2=14$ at the point (3, 2, 1).

(c) Evaluate $\int_C \vec{F} . d\vec{r}$ along the curve $x^2 + y^2 = 1$, $z = 1$ in the positive direction from $(0, 1, 1)$ to $(1, 0, 1)$, where :

$$\vec{F} = (yz + 2x)\hat{i} + xz\hat{j} + (xy + 2z)\hat{k}.$$

7. (a) Find the characteristic equation of the matrix :

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

and hence find the matrix represented by

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I,$$

where $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(b) Investigate for what values of λ, μ the simultaneous equations :

$x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$

have (i) no solution, (ii) a unique solution (iii) an infinite number of solutions.

(c) If $N=\begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$ is a matrix, then show that $(I-N)(I+N)^{-1}$ is unitary matrix, where $I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.